Monopolistic Competition: CES Redux?*

Paolo Bertoletti\textsuperscript{1}  
Pavia University and IEFE

Paolo Epifani\textsuperscript{3}  
Bocconi University and BAFFI

February 2014

Abstract

We study the competitive and reallocation effects of trade opening in monopolistic competition. To this purpose, we generalize the Melitz (2003) setup with heterogeneous firms and fixed and variable trade costs beyond the CES to the case of additively separable utility functions. We find that extensive margin (Melitz-type selection) effects are robust to relaxing the CES assumption. Intensive margin effects (market share reallocations across inframarginal firms) and competitive (markup) effects are instead fragile. An important implication is that measured productivity gains from trade opening are no longer ensured with non-CES preferences. We discuss our results in the light of alternative setups featuring non-additive preferences, strategic interaction and consumers’ preference for an ideal variety.

\textit{JEL Classification:} F1; \textit{Keywords:} Monopolistic Competition; CES Preferences; International Trade; Competitive and Reallocation Effects.

1 Introduction

In this paper, we explore the robustness of the competitive and reallocation effects of trade opening in monopolistic competition. Our study is motivated by the fact that the Dixit and Stiglitz (1977, henceforth D-S) setup with constant elasticity of substitution (CES) preferences, so far the workhorse of trade economists, has been put under fire by a recent

\textsuperscript{*}The paper has benefited enormously from the comments of Bob Staiger (the Editor) and two anonymous referees. We are also grateful to Kristian Behrens, Fabrice Dehever, Swati Dhingra, Federico Etro, Eileen Fumagalli, Gino Garcia, Dennis Novy, Marco Ottaviani, Marcello Pagnini, Clara Poletti, Luca Salvatici, Piero Tedeschi and seminar participants at many venues for helpful comments and discussions. All errors are our own.

\textsuperscript{1}Department of Economics and Management, Università di Pavia, Via San Felice 5, 27100 Pavia (Italy), and IEFE, Bocconi University. E-mail: paolo.bertoletti@unipv.it

\textsuperscript{3}Department of Economics and BAFFI, Università Bocconi, Via Röntgen 1, 20136 Milan (Italy). E-mail: paolo.epifani@unibocconi.it.
The traditional critique that CES preferences make monopolistic competition little interesting, due to the implied invariance of markups to trade opening, has been recently revived by Neary (2004, 2009a, 2010), who proposes to study competitive effects in oligopoly. Dhingra and Morrow (2012, henceforth DM) and Zhelobodko, Kokovin, Parenti and Thisse (2012, henceforth ZKPT) propose instead to relax the CES assumption in the standard D-S setup. In particular, ZKPT shows that CES preferences are a knife-edge between cases yielding opposite competitive and selection effects, thereby concluding that a theory of monopolistic competition cannot be built on CES preferences, due to the "peculiarity" of its implications. We argue instead that the critique of the CES assumption in monopolistic competition has gone too far. First, because although CES preferences are clearly peculiar, relaxing the CES assumption does not obviously lead to less peculiar competitive and reallocation effects. Second, because selection effects à la Melitz (2003) are indeed robust to relaxing the CES assumption.

In Section 2, we generalize the Melitz (2003) model to the case of additively separable utility functions by exploiting his approach of relating the behavior of active firms to that of a cutoff firm. In Section 3, we then use the model to study the competitive and reallocation effects of trade opening, both in the stylized scenario of pure globalization (formally equivalent to an increase in market size) and in the more general case of costly trade. Our approach allows us to easily obtain some of the results independently found by ZKPT and DM, to identify a new source of reallocations and to deal with the case of costly trade. In particular, we show that reallocation effects consist of two distinct mechanisms: an extensive margin (or Melitz-type selection) effect, whereby trade changes the set of active firms through entry and exit, and an intensive margin effect, whereby trade affects the market shares of inframarginal firms. By competitive effects we refer instead to the impact of trade on firms’ price-marginal cost markups. We show how, in non-CES environments, the effects of trade opening depend on properties of the additive functional forms that we characterize in this paper, and how the latter affect aggregate productivity.

Our main results can be summarized as follows. When trade costs are large enough to induce a partitioning of firms into exporters and non-exporters (arguably, the empiri-
ally relevant case), extensive margin effects are robust to relaxing the CES assumption. Intensive margin and competitive effects are instead fragile. An important implication is that measured productivity gains from trade opening are not ensured with non-CES preferences. We argue that these results arise from the fact that, in a D-S setup with additive preferences (with or without heterogeneous firms), markup changes do not fully reflect the action of competitive forces. In Section 4, we therefore consider alternative monopolistic competition environments. We first consider the example of quasi-linear quadratic preferences, as in Melitz and Ottaviano (2008), thereby relaxing the assumption of additive preferences. Next, we relax the D-S assumption that firms do not interact strategically by studying both Bertrand and Cournot competition with additive preferences. Finally, we reconsider Lancaster’s (1979) ideal variety approach to monopolistic competition. Surprisingly, we find that none of these setups delivers a strong and robust trade-induced pro-competitive mechanism. In Section 5, we briefly conclude that one possible interpretation of our results is that they indirectly support the venerable D-S monopolistic competition setup with CES preferences.

Our paper is related to the vast theoretical literature on monopolistic competition and international trade, initiated by Dixit and Stiglitz (1977), Krugman (1979, 1980), Lancaster (1979), Helpman (1981), and whose early contributions are systematized in Helpman and Krugman (1985). It is also closely related to Melitz (2003), whose methodology and results we generalize to the case of additively separable utility functions, and to a recent literature which independently studies some implications of non-CES preferences in a D-S monopolistic competition setup with heterogeneous firms. In particular, our setup is similar but more general than that of ZKPT, which leads us to reach different conclusions. Specifically, ZKPT does not study the intensive margin effects of trade opening and their impact on aggregate productivity, nor the cases of costly trade and strategic interactions. DM studies the welfare effects of an increase in market size, and is therefore complementary to our paper, as we do not focus on welfare issues. Arkolakis, Costinot, Donaldson and Rodriguez-Clare (2012, henceforth ACDR) is also complementary to our work, as it shows some counter-intuitive welfare implications of variable markups in monopolistic competition. Finally, Mrázová and Neary (2012) complements our analysis by introducing a distinction between first-order and second-order selection effects (the former involve a binary choice between performing or not a certain activity, whereas the latter concern how to perform a certain activity) and showing that the former are more robust

\[ \text{In particular, ACDR shows that, with a Pareto distribution of firm productivity and preferences featuring log concavity of demand and a choke-off price, the welfare gains from a reduction in trade costs are smaller than those predicted by models with constant markups.} \]
than the latter.

2 Setup

In this Section, we illustrate the Dixit-Stiglitz monopolistic competition setup with additive preferences and heterogeneous firms. In the next Section, we will use it to study the competitive and reallocation effects of trade opening.

**Demand.** Consider an economy populated by \( L \) consumers/workers, whose wage is \( w = 1 \). They share the same additively separable and symmetric preferences, defined over a continuum of varieties and represented by the utility function

\[
U = \int_0^N u(c_i) \, di,
\]

where \( c_i \) is individual consumption of variety \( i \), and \( N \) is the mass of potential varieties. Only \( n < N \) varieties are available for consumption. The sub-utility function \( u(\cdot) \) is strictly increasing and concave, and is at least thrice continuously differentiable. In particular, we assume that \( u'(c) > 0 > u''(c) \) for \( c > 0 \), and \( u(0) = 0 \).

Utility maximization subject to the budget constraint \( \int_0^n p_i c_i \, di \leq 1 \), where \( p_i \) is the price of variety \( i \), yields \( u'(c_i) = \lambda p_i \), where \( \lambda = \int_0^n u'(c_i) c_i \, di \) is the marginal utility of income. Under the D-S assumption that firms treat \( \lambda \) as a constant, the inverse individual demand for a variety is thus

\[
p(c) = \frac{u'(c)}{\lambda},
\]

where we have dropped the variety index, as all firms face the same demand function. The price elasticity of demand, \( \varepsilon(c) \), is given by

\[
\varepsilon(c) = -\frac{p(c)}{p'(c)c} = -\frac{u'(c)}{u''(c)c},
\]

and can be shown to equal the elasticity of substitution between any two varieties, \( \sigma(c) \), when they are consumed in the same amount \( c \).

\(^{4}\) Krugman (1979) assumed that \( \sigma'(c) < 0 \), i.e., a decreasing elasticity of substitution (henceforth, DES preferences); \( \sigma'(c) > 0 \) represents instead the case of an increasing elasticity of substitution (henceforth, IES preferences) and \( \sigma'(c) = 0 \) is the well-known CES case.\(^{5}\)

\(^{4}\) Note that \( \varepsilon \) is independent of \( \lambda \), and that it equals the reciprocal of the elasticity of marginal utility with respect to individual consumption. See also Blackorby and Russell (1989) on the elasticity of substitution.

\(^{5}\) As far as we know, the case of monopolistic competition with IES preferences was first studied by Bertoletti, Poletti and Fumagalli (2008). Bertoletti (2006) and Behrens and Murata (2012) discuss instead a specific functional form for DES preferences.
Production. Producing a differentiated variety involves a fixed cost $\alpha$ and a marginal cost $\beta$. As in Melitz (2003) we assume that, upon paying a fixed entry cost $e$, firms draw their marginal cost $\beta \in [\bar{\beta}, \infty)$ from a common, continuous cumulative distribution $G(\beta)$ with density $g(\beta)$ and $\bar{\beta} > 0$. All costs are in terms of labor.

Using (1), firm revenue equals $p(c)\lambda L = r(c)(L/\lambda)$, where $r(c) = u'(c)c$. Marginal revenue and the derivative of marginal revenue therefore equal, respectively, $r'(c)/\lambda$ and $r''(c)/(\lambda L)$, where

$$r'(c) = u'(c) + u''(c)c, \quad r''(c) = 2u''(c) + u'''(c)c. \quad (3)$$

The first-order condition for profit maximization yields:

$$r'(c) = \lambda \beta, \quad (4)$$

where we assume that $r'(c) > 0 > r''(c)$ for all $c > 0$ to obtain a well-behaved solution.$^6$ Note that $r' > 0$ implies $\sigma > 1$.

Equations (1) and (4) imply that the profit-maximizing price rule can be written as

$$p = m(c)\beta, \quad m(c) = \frac{u'(c)}{r'(c)} = \frac{\sigma(c)}{\sigma(c) - 1}, \quad (5)$$

where $m(c)$ is the price-marginal cost markup. Note that $m' \geq 0 \iff \sigma' \leq 0$, and that

$$\sigma'(c) \geq 0 \iff \frac{r'(c)u''(c)}{r''(c)u'(c)} = \frac{\eta(c)}{\sigma(c)} \geq 1, \quad \eta(c) = -\frac{r'(c)}{r''(c)c} > 0,$$

where $\eta(c)$ is the reciprocal of the elasticity of marginal revenue in absolute value. Thus, markups are increasing in individual consumption $c$ with DES preferences ($\sigma' < 0 \iff \eta < \sigma$), decreasing in $c$ with IES preferences ($\sigma' > 0 \iff \eta > \sigma$) and constant in the CES case ($\sigma = \eta$). Note also, for later reference, that the elasticity of $m(c)$ can be written as:

$$\frac{d \ln m(c)}{d \ln c} = \frac{1}{\eta(c)} - \frac{1}{\sigma(c)}. \quad (6)$$

Finally, using (5), the expression for variable profits $\pi_v$ can be written as:

$$\pi_v(\beta, c, L) = [m(c) - 1] \beta L c = \left[\frac{u'(c)}{r'(c)} - 1\right] \beta L c. \quad (7)$$

---

$^6$We also assume that an interior solution to the problem of profit maximization exists. Sufficient (but not necessary) conditions are: $\lim_{c \to \infty} r'(c) = 0$ and $\lim_{c \to 0} r'(c) = \infty$. 

5
Differentiating (7) yields that $\pi_v$ is increasing in $c$, with
\[
\frac{\partial \ln \pi_v}{\partial \ln c} = \frac{\sigma(c)}{\eta(c)} > 0. \tag{8}
\]

**Individual consumption.** Denote by $\beta^*$ the marginal cost cutoff, namely, the value of $\beta$ satisfying the zero cutoff profit condition $\pi(\beta^*, c^*, L, \alpha) = 0$, where $c^*$ is individual consumption of a cutoff firm’s good, and $\pi(\beta, c, L, \alpha) = \pi_v(\beta, c, L) - \alpha$ is total profit. Thus, using (7),
\[
\pi_v(\beta^*, c^*, L) = [m(c^*) - 1] \beta^* c^* L = \alpha, \tag{9}
\]
which implicitly defines $c^*(\beta^*, L, \alpha)$. Following Krugman (1979) and Melitz (2003), we can now relate the behavior of an active $\beta$-firm to that of a cutoff firm by eliminating the marginal utility of income $\lambda$ from the relevant first-order conditions for profit maximization. Formally, $r'(c^*) = \lambda \beta^*$ by (4). Solving for $\lambda$ and substituting back into (4) yields:
\[
r'(c^*) = r'(c^*) \frac{\beta^*}{\beta}. \tag{10}
\]
Eq. (10) is key to the characterization of the equilibrium, as it implicitly defines the individual consumption of a $\beta$-firm’s good as $c(\beta; \beta^*, c^*)$. Using $c(\beta; \beta^*, c^*(\beta^*, L, \alpha))$, we can then show how $c(\beta; \beta^*, c^*(\beta^*, L, \alpha))$ varies with $\beta^*$, $L$ and $\alpha$. In the Appendix, we prove the following

**Lemma 1** Individual consumption of a $\beta$-firm’s good, $c(\beta; \beta^*, L, \alpha)$, is increasing in $\beta^*$ and $\alpha$ and decreasing in $L$, with
\[
\frac{\partial \ln c}{\partial \ln \beta^*} = \frac{\eta(c)}{m(c^*)} > 0, \quad \frac{\partial \ln c}{\partial \ln L} = -\frac{\partial \ln c}{\partial \ln \alpha} = -\frac{\eta(c)}{\sigma(c^*)} < 0.
\]

**Free entry.** Free entry implies that expected profits, $\pi^E$, equal the fixed entry cost, $\alpha_e$:
\[
\pi^E = \int_\beta^{\beta^*} \pi(\beta, c, L, \alpha)dG(\beta) = \alpha_e, \tag{11}
\]
where $c = c(\beta; \beta^*, L, \alpha)$. Differentiating $\pi^E$ with respect to $\beta^*$, using (8) and Lemma 1, gives:
\[
\frac{\partial \ln \pi^E}{\partial \ln \beta^*} = \int_\beta^{\beta^*} \frac{\partial \ln \pi_v}{\partial \ln c} \frac{\partial \ln c}{\partial \ln \beta^*} dG(\beta) = \int_\beta^{\beta^*} \frac{\sigma(c)}{m(c^*)} dG(\beta) > 0. \tag{12}
\]

Note, from (9), that this and the following results actually depend on $L/\alpha$. For expositional purposes, however, in this Section we treat $L$ and $\alpha$ as separate arguments.
Thus, as in Melitz (2003), expected profits are increasing in $\beta^*$ and the free-entry condition (11) uniquely pins down $\beta^*$ as a function of $L$, $\alpha$ and $\alpha_c$, thereby defining the equilibrium value of $c(\beta; L, \alpha, \alpha_c)$.

**Firm size, profits and markups.** From now on, we index firms by their marginal cost $\beta$ and denote by $c(\beta)$ the equilibrium individual consumption of a $\beta$-firm’s good. Let $m(\beta) = m(c(\beta))$, $p(\beta) = m(\beta)\beta$ and $\pi_v(\beta) = [m(\beta) - 1] \beta c(\beta)L$ be the optimal markup, price and variable profits of a $\beta$-firm.\(^8\) As shown in the Appendix:

$$
\frac{d\ln c(\beta)}{d\ln \beta} = -\eta(c(\beta)) < 0, \quad \frac{d\ln p(\beta)c(\beta)}{d\ln \beta} = 1 - \chi(c(\beta)) < 0, \quad (13)
$$
$$
\frac{d\ln \pi_v(\beta)}{d\ln \beta} = 1 - \sigma(c(\beta)) < 0, \quad \frac{d\ln m(\beta)}{d\ln \beta} = \frac{\eta(c(\beta))}{\sigma(c(\beta))} - 1 \geq 0 \Leftrightarrow \sigma'(c) \geq 0, \quad (14)
$$

where

$$
\chi(c) = 1 + \eta(c) \left[ 1 - \frac{1}{\sigma(c)} \right] > 1, \quad \chi(c) \geq \sigma(c) \Leftrightarrow \sigma'(c) \geq 0. \quad (15)
$$

Note that, as in Melitz (2003), high-productivity (low-$\beta$) firms are larger, both in terms of output and revenue, and more profitable. Unlike the Melitz model, however, where preferences are CES and markups are constant, in this setup high-productivity firms charge higher markups with DES preferences and lower markups with IES preferences. Moreover, $\eta = \sigma = \chi$ for $\sigma' = 0$. With non-CES preferences, instead, $\eta \neq \sigma \neq \chi$ and all these variables are functions of $c$, thereby implying that output, revenue and profit profiles are governed, respectively, by $\eta$, $\chi$ and $\sigma$.

### 3 Competitive and Reallocation Effects of Trade Opening

Having laid down the setup of the model, we can now study the extensive margin, intensive margin and competitive effects of trade opening. We begin by analyzing the impact of an increase in market size $L$, which can be interpreted as pure globalization. This will allow us to clarify the basic forces at work in our setup and to better compare our results to previous works. Then we will study the more realistic case of costly trade, in which trade opening leads to a reduction in fixed or variable trade costs.

---

\(^8\)The measure of active firms $n$ is determined by the budget condition, requiring average expenditure to equal $1/n$. Thus,

$$
n = \left[ \int_0^{\beta^*} p(\beta)c(\beta) \frac{dG(\beta)}{G(\beta')} \right]^{-1}.
$$
3.1 Pure Globalization

**Extensive margin effects.** The extensive margin (selection) effects of pure globalization are captured by the impact of an increase in \( L \) on the marginal cost cutoff \( \beta^* \). As shown in the Appendix, \( d\beta^*/dL \gtrless 0 \Leftrightarrow \sigma' \gtrless 0 \). Thus, pure globalization leads to a standard selection effect with DES preferences. With IES preferences, instead, it leads to an anti-selection effect, whereby less productive firms can survive in a larger market.\(^9\) The mechanics behind these results can be summarized as follows. First, using Lemma 1, the total impact of an increase in \( L \) on individual consumption equals

\[
\frac{d \ln c(\beta)}{d \ln L} = \frac{\partial \ln c}{\partial \ln L} + \frac{\partial \ln c}{\partial \ln \beta^*} \frac{d \ln \beta^*}{d \ln L} = -\eta(c(\beta))\epsilon_{\lambda,L},
\]

where \( \epsilon_{\lambda,L} = 1/\sigma(c^*) - (1/m(c^*)) \) \( (d \ln \beta^*/d \ln L) \) is a term common to all firms. Next, using (8) and (16), the total impact of an increase in market size on variable profits equals

\[
\frac{d \ln \pi_v(\beta)}{d \ln L} = 1 + \frac{\partial \ln \pi_v}{\partial \ln c} \frac{d \ln c}{d \ln L} = 1 - \sigma(c(\beta))\epsilon_{\lambda,L}.
\]

Finally, totally differentiating \( \pi^E \), and using (17), yields:

\[
\int_{\beta}^{\beta^*} \frac{d \ln \pi_v}{d \ln L} dG(\beta) = \int_{\beta}^{\beta^*} \left[ 1 - \sigma(c(\beta))\epsilon_{\lambda,L} \right] dG(\beta) = 0,
\]

which implies that \( \epsilon_{\lambda,L} > 0 \), and hence that \( c(\beta) \) is decreasing in \( L \). The intuition is that, ceteris paribus, an increase in market size raises variable profits proportionally by increasing all market demands; individual demands must therefore fall to restore the zero expected profit condition. Note, also, that \( \epsilon_{\lambda,L} = 1/\sigma \) and \( d \ln \pi_v(\beta)/d \ln L = 0 \) in the CES case. With non-CES preferences, instead, the sign of \( d \ln \pi_v(\beta)/d \ln L \) depends on \( \beta \): when \( \sigma' < 0 \), a rise of \( L \) leads to an increase in the relative profits of more productive firms. In this case, the profits of a \( \beta^* \)-firm become negative and the marginal cost cutoff must fall to restore the zero cutoff profit and free entry conditions. Eq. (18) also implies that the profits of more productive firms increase. Conversely, when \( \sigma' > 0 \), an increase in \( L \) raises the relative profits of less productive firms, and thus leads to a rise of \( \beta^* \) and a reduction in the profits of more productive firms.

To conclude, as also argued by ZKPT and DM, selection effects seem to crucially depend on the assumptions about preferences. In the next Section we show, however,

---

\(^9\)It is also possible to show that, independent of consumer preferences, an increase in the fixed production cost \( \alpha \) leads to a selection effect, whereas an increase in the fixed entry cost \( \alpha_e \) leads to an anti-selection effect.
that the fragility of Melitz-type selection effects is more apparent than real, as the above results hold only in the absence of trade frictions.

**Intensive margin effects.** ZKPT and DM seem to argue that selection effects are the only source of reallocations induced by an increase in $L$. Note, however, that with non-CES preferences an increase in market size also reallocates market shares (both in terms of sales and revenue) across inframarginal firms. This follows immediately from (16), which implies that a rise of $L$ leads to an increase in the relative output of high-productivity firms when $\eta' < 0$, and to an increase in the relative output of low-productivity firms when $\eta' > 0$. Similarly, using (6) and (16), the total impact of $L$ on $p(\beta)c(\beta)$ equals:

$$\frac{d \ln \{p(\beta)c(\beta)\}}{d \ln L} = \left(\frac{d \ln m}{d \ln c} + 1\right) \frac{d \ln c}{d \ln L} = -\chi'(c(\beta)) \epsilon_{\lambda,L} < 0.$$  

Evidently, an increase in market size leads to an increase in the relative revenue of high-productivity firms when $\chi' < 0$, and to an increase in the relative revenue of low-productivity firms when $\chi' > 0$. We summarize our main results in the following

**Proposition 1** The extensive margin effects of pure globalization are governed by the sign of $\eta'$. Instead, the intensive margin effects in terms of sales and revenue are governed, respectively, by the signs of $\eta'$ and $\chi'$.

Note also that:

$$\eta'(c) = -\frac{r''(c)c - [r'''(c)c + r''(c)]r'(c)}{(r''(c)c)^2} \Rightarrow \eta'(c) \geq 0 \iff -\frac{r'''(c)c}{r''(c)c} \geq \frac{1 + \eta(c)}{\eta(c)},$$

where $r''' = u'''c + 3u''$. Moreover, differentiating $\chi(c)$ yields:

$$\chi'(c) = \frac{\eta(c)}{\sigma(c)} \left[ \frac{\eta'(c)}{\eta(c)} (\sigma(c) - 1) + \frac{\sigma'(c)}{\sigma(c)} \right].$$

Consequently, the intensive margin effects seem to depend on properties of preferences that are hard to pin down and are not determined by the sign of $\sigma'$. As discussed in the

---

10The marginal utility of income, $\lambda$, provides an alternative way of illustrating the model mechanics. To see how, note once again that, ceteris paribus, an increase in market size raises variable profits proportionally by increasing all market demands. By (1), $\lambda$ must therefore increase to reduce individual demands and profits. Note also, from (4), that a rise of $L$ can affect the profit-maximizing level of $c(\beta)$ only through $\lambda$, and that $\lambda$ and $\beta$ play a symmetric role in determining $c(\beta)$. Therefore, the results in (13) and (14) immediately tell us that, with non-CES preferences, the effects of an increase in market size $L$ on relative firm output, revenue and profits are determined by the signs of $\eta'$, $\chi'$ and $\sigma'$. Actually, it is easily shown that:

$$\frac{d \ln c(\beta)}{d \ln L} = \frac{d \ln c}{d \ln \lambda} \frac{d \ln \lambda}{d \ln L} = -\eta(c(\beta)) \frac{d \ln \lambda}{d \ln L},$$

thereby implying that $\epsilon_{\lambda,L}$ is the elasticity of $\lambda$ with respect to $L$.  

9
Appendix, an important implication is that pure globalization has an ambiguous impact on a measure of aggregate productivity when the implied extensive and intensive margin effects point in opposite directions. For instance, the selection effect implied by $\sigma' < 0$ may be associated with a reduction in measured productivity if $\eta' > 0$.\footnote{Although measured productivity changes do not directly measure welfare changes, they provide the standard way of quantifying empirically the gains from trade due to reallocations across heterogeneous firms: see, e.g., Melitz and Treffer (2012, p. 109) on this point.}

We record this result in the following

**Remark 1** If $\sigma'$ and $\eta'$ do not agree in sign, a selection effect may be associated with a reduction in measured productivity when preferences are DES; an anti-selection effect may instead be associated with an increase in measured productivity when preferences are IES.

**Competitive effects.** The competitive effects of pure globalization depend on how it affects the behavior of inframarginal firms and the selection of firms through entry or exit. The impact on inframarginal firms entirely depends on how an increase in $L$ affects individual consumption. Since $c$ is decreasing in $L$, an increase in market size leads inframarginal firms to charge lower markups when $\sigma' < 0$ and higher markups when $\sigma' > 0$.\footnote{Similarly, it is possible to show that an increase in $\alpha$ or $\alpha_e$ are pro-competitive with IES preferences and anti-competitive with DES preferences.} Moreover, the anti-competitive effect associated with IES preferences is strengthened by the concomitant anti-selection effect, as the latter implies entry of low-productivity, high-markup firms. In contrast, with DES preferences there is a countervailing increase in average markups due to exit of low-productivity, low-markup firms. Finally, the impact on the average markup weighted by market shares also depends on the intensive margin effects discussed above.

### 3.2 Costly Trade

We now show how the effects of a reduction in trade cost are different from those of pure globalization. To this purpose, we consider two identical countries separated by symmetric trade costs. Variables related to the foreign market will be indexed by an $x$, whereas variables related to the domestic market will not be indexed. Selling in the foreign market involves a fixed cost of exporting $\alpha_x \geq 0$ and a variable iceberg trade cost $\tau \geq 1$. As in Melitz (2003), we focus on the more interesting case in which trade costs are large enough to induce a partitioning of firms into exporters and non-exporters. This requires
\[ \beta^* > \beta_x^*, \text{ where } \beta_x^* \text{ is the marginal cost cutoff for exporting firms.}^{13} \]

A \( \beta \)-firm’s profits in the domestic and foreign markets are given, respectively, by
\[ \pi(\beta) = \pi_d(\beta) - \alpha \text{ and } \pi_x(\beta) = \pi_v(\tau \beta) - \alpha_x, \]
where \( \pi_v(\beta) = [m(c(\beta)) - 1] \beta c(\beta) L, \) \( L \) is the size of each market, and \( \tau^{-1}(\alpha_x) \leq \tau \pi_v^{-1}(\alpha) \) to satisfy the condition \( \beta^* \geq \beta_x^*. \)

Thus, the marginal cost cutoff for exporters is implicitly given by:
\[ \pi_x(\beta^*_x) = [m(c(\tau \beta_x^*))] - 1] \tau \beta_x^* c(\tau \beta_x^*) L - \alpha_x = 0. \quad (19) \]

Finally, the free-entry condition is now given by:
\[ \pi^E = \int_\frac{\beta}{2}^{\beta^*} \pi(\beta)dG(\beta) + \int_\frac{\beta}{2}^{\beta^*_x} \pi_x(\beta)dG(\beta) = \alpha_e. \quad (20) \]

We use (19) to define \( \beta^*_x(\beta^*, \tau, \alpha_x) \), and (9), (10) and (20) to pin down \( \beta^* \).\(^{15}\)

**Extensive margin effects.** As shown in the Appendix, applying the implicit function theorem to (20) yields:
\[ \frac{\partial \ln \beta^*}{\partial \ln \tau} = \frac{m(c^*) \int_\frac{\beta}{2}^{\beta^*_x} \tau \beta c(\tau \beta) LdG(\beta)}{\int_\frac{\beta}{2}^{\beta^*} p(\beta) c(\tau \beta) LdG(\beta) + \int_\frac{\beta}{2}^{\beta^*_x} p(\tau \beta) c(\tau \beta) LdG(\beta)} > 0. \quad (21) \]

Evidently, a reduction in \( \tau \) leads to a reduction in \( \beta^* \), a standard selection effect. Moreover, in the Appendix we show that \( \partial \beta^*/\partial \alpha_x > 0 \), i.e., that a reduction in the fixed cost of exporting \( \alpha_x \) also leads to a selection effect.

The intuition for why extensive margin effects are robust to the sign of \( \sigma' \) is that, as in the Melitz model, a reduction in trade costs increases expected profits by increasing exporting firms’ profits at the expense of non-exporting firms, which face instead a tougher competition in the domestic market. Consequently, a cutoff firm’s profits become negative

\(^{13}\)Given that we consider two identical countries, balanced trade requires the wage to be equalized, and we can use it as the numeraire. Moreover, our strategy of eliminating the (common) marginal utility of income proves as useful as in the case of pure globalization.

\(^{14}\)Note that a domestic firm producing only for the foreign market would incur an overall fixed cost equal to \( \alpha + \alpha_x \), which implies that this case cannot arise in equilibrium. For analytical convenience, we can therefore apportion the fixed cost \( \alpha \) to domestic profits, in this following the heterogeneous-firm literature.

\(^{15}\)The measure of active firms is given by:
\[ n = \left[ \int_\frac{\beta}{2}^{\beta^*} p(\beta) c(\beta) \frac{dG(\beta)}{G(\beta)} + \int_\frac{\beta}{2}^{\beta^*_x} p(\tau \beta) c(\tau \beta) \frac{dG(\beta)}{G(\beta \tau)} \right]^{-1}. \]
Proposition 2 When fixed \((\alpha_x)\) and variable \((\tau)\) trade costs induce a partitioning of firms into exporters and non-exporters, a reduction in either \(\alpha_x\) or \(\tau\) leads to a reduction in \(\beta^*\). The extensive margin effects of a reduction in fixed and variable trade costs are therefore robust to the sign of \(\sigma'\).

Intensive margin effects. Consider first a reduction in the fixed cost of exporting \(\alpha_x\). Note that \(\alpha_x\) affects individual consumption only through \(\beta^*\), and that \(\beta^*\) is increasing in \(\alpha_x\) according to Proposition 2. Moreover, individual consumption is increasing in \(\beta^*\) according to Lemma 1. It follows that a reduction in \(\alpha_x\) reduces both \(c = c(\beta)\) and \(c_x = c(\tau\beta)\), with:

\[
\frac{d \ln c(\beta)}{d \ln \alpha_x} = \frac{\eta(c(\beta))}{m(c^*)} \frac{\partial \ln \beta^*}{\partial \ln \alpha_x} > 0, \quad \frac{d \ln c(\tau\beta)}{d \ln \alpha_x} = \frac{\eta(c(\tau\beta))}{m(c^*)} \frac{\partial \ln \beta^*}{\partial \ln \alpha_x} > 0. \tag{22}
\]

In both expressions, \(\eta\) is the only \(\beta\)-firm specific term. Consequently, as in the case of pure globalization, the intensive margin effects of a reduction in \(\alpha_x\) depend on the sign of \(\eta'\).\(^{16}\)

Consider now a reduction in the variable trade cost \(\tau\). By Proposition 2 and Lemma 1, this leads to a reduction in \(\beta^*\) and thus in \(c\):

\[
\frac{d \ln c(\beta)}{d \ln \tau} = \frac{\eta(c(\beta))}{m(c^*)} \frac{\partial \ln \beta^*}{\partial \ln \tau} > 0. \tag{23}
\]

As for \(c_x\), a reduction in \(\tau\) has both a positive direct effect and a negative indirect effect through \(\beta^*\). Using Lemma 1 and (13) we can write:

\[
\frac{d \ln c(\tau\beta)}{d \ln \tau} = \frac{\partial \ln c}{\partial \ln \tau} + \frac{\partial \ln c}{\partial \ln \beta^*} \frac{\partial \ln \beta^*}{\partial \ln \tau} = -\eta(c(\tau\beta)) \left(1 - \frac{1}{m(c^*)} \frac{\partial \ln \beta^*}{\partial \ln \tau}\right) < 0, \tag{24}
\]

where the inequality follows directly from (21). Thus, (23) and (24) imply that a reduction in \(\tau\) reduces individual consumption of domestic varieties \(c\) and increases individual consumption of foreign varieties \(c_x\). Moreover, in both expressions \(\eta\) is the only \(\beta\)-firm specific term. Consequently, independent of the sign of \(\eta'\), a reduction in \(\tau\) affects the relative output of inframarginal firms in opposite directions in the domestic and foreign market. For instance, if \(\eta' < 0\), a reduction in \(\tau\) leads to an increase in the relative sales of high-productivity firms in the domestic market, and to a reduction in the relative sales

\(^{16}\)For brevity, in this Section we focus on intensive margin effects in terms of sales.
of high-productivity inframarginal exporters in the foreign market.\footnote{A reduction in trade costs can also be shown to increase $\beta^*_x$, thereby increasing the set of exporters.} Just the opposite results hold for $\eta' > 0$. We record these results in the following

**Proposition 3** The intensive margin effects (in terms of sales) of a reduction in fixed and variable trade costs are governed by the sign of $\eta'$. A change in the variable trade cost $\tau$ leads to opposite intensive margin effects in the domestic and foreign market.

**Competitive effects.** According to (22), a reduction in $\alpha_x$ reduces both $c$ and $c_x$, thus leading inframarginal firms to charge lower markups with DES preferences and higher markups with IES preferences. Conversely, by (23) and (24), a reduction in $\tau$ reduces $c$ and increases $c_x$, leading inframarginal firms to charge lower markups in the domestic market and higher markups in the export market when preferences are DES. The opposite result holds instead when preferences are IES.\footnote{As discussed in the previous Section, also the extensive margin effects impact on average markups.}

Finally, note that

$$
\left[ \frac{d \ln c(\beta)}{d \ln \tau} + \frac{d \ln c(\tau \beta)}{d \ln \tau} \right]_{\tau=1} = \left[ \eta(c(\beta)) \left( \frac{\partial \ln \beta^*}{\partial \ln m(c^*)} - 1 \right) \right]_{\tau=1} < 0, \tag{25}
$$

where the inequality follows from (21), which implies that $\partial \ln \beta^*/\partial \ln \tau < m(c^*)/2$ when evaluated at $\tau = 1$. Thus, by continuity, a small increase in $\tau$ around $\tau = 1$ leads all inframarginal exporting firms to reduce average sales, and therefore to charge a lower average markup across markets with DES preferences (a higher markup with IES preferences).\footnote{Equation (25) also implies that, with non-CES preferences, average markups may be non-monotonically related to trade costs. This follows from the fact that a move from costless trade to autarky is equivalent to a reduction in market size and therefore leads firms to charge higher markups with DES preferences (and lower markups with IES preferences), as discussed in the previous Section. We record these results in the following}

Equation (25) also implies that, with non-CES preferences, average markups may be non-monotonically related to trade costs. This follows from the fact that a move from costless trade to autarky is equivalent to a reduction in market size and therefore leads firms to charge higher markups with DES preferences (and lower markups with IES preferences), as discussed in the previous Section. We record these results in the following

**Proposition 4** With DES preferences, a reduction in the fixed cost of exporting $\alpha_x$ leads
inframarginal firms to charge lower markups in the domestic and foreign market. Instead, a reduction in the variable trade cost $\tau$ leads inframarginal firms to charge lower markups in the domestic market and higher markups in the foreign market. The opposite results hold with IES preferences. In both cases, the average markup of inframarginal firms is non-monotonically related to $\tau$.

3.3 Discussion

Our results suggest that, although the extensive margin effects of trade opening are robust to relaxing the CES assumption, the intensive margin and competitive effects are instead fragile. An important implication is that gains from trade opening in terms of measured productivity are no longer warranted in our setup.

The reason why allowing for variable markups in the Melitz model opens up a Pandora’s box of contrasting and possibly counter-intuitive reallocation effects is closely related to the nature of competitive effects in a Dixit-Stiglitz setup (with or without heterogeneous firms), in which markups are entirely driven by the level of individual consumption $c$. This turns out to be restrictive for many reasons. The first is that the relationship between individual consumption and markups is not robust to the assumptions about preferences: markups are increasing in individual consumption with DES preferences and decreasing with IES preferences. This explains why pure globalization is largely pro-competitive with DES preferences and anti-competitive with IES preferences.

Second, the equilibrium level of individual consumption is naturally decreasing in the marginal cost, which leads to a pass-through effect whereby markups vary with marginal costs. For instance, markups are decreasing in marginal costs with DES preferences (incomplete pass-through), which explains why a reduction in the marginal cost of exporting leads to higher markups in the foreign market. This also explains why a genuinely pro-competitive shock, such as a reduction in variable trade costs, may have a non-monotonic effect on average markups. Finally, it implies that, whatever the assumptions about preferences, a reduction in variable trade costs leads to opposite intensive margin effects in

---

20These peculiar implications are perhaps surprising also because the IES assumption is *prima facie* no less reasonable than the CES assumption. Indeed, a possible rationalization of IES preferences is that, by their very nature, differentiated varieties of some product can be used to perform either generic or more specific tasks. For instance, a blue pencil can be used either to write down a laundry list (for which a red pencil would be equally appropriate) or, jointly with a red pencil, to mark different types of comments on an exam paper. Hence, a fall in the symmetric endowment of varieties, by reducing the opportunity to use varieties to perform specific tasks, may also reduce their substitutability. Moreover, consider a situation in which you are endowed with two red pencils and two blue pencils, and compare it with a situation in which you are endowed instead with ten red and ten blue pencils: if you perceive a red and a blue pencil as more substitutable in the latter situation, then your preferences are IES. See however Mrázová and Neary (2012) for an argument in support of DES preferences which appeals to the so-called Marshall’s Second Law of Demand.
the domestic and foreign market.

The direct dependence of markups only on individual consumption implies that the effects of trade opening are partly unrelated to pro-competitive forces. To deepen these issues, in the next Section we will explore the robustness of competitive effects in alternative monopolistic competition environments.

4 Alternative Environments

One possible explanation for the lack of robust competitive effects in a D-S monopolistic competition setup with additive preferences is that the latter implies that markups do not directly depend on either the behavior of competitors or on their number $n$. This is because the Dixit-Stiglitz assumption that firms do not interact strategically implies that the demand elasticity that they perceive equals the elasticity of substitution, and the latter is independent of the number of varieties when preferences are additive.\textsuperscript{21} Moreover, in this setup the number of product characteristics equals the number of varieties/firms and thus an increase in $n$ does not "crowd" the variety space.\textsuperscript{22} It follows that a pro-competitive effect can arise only from variation in $c$, which does not fully capture, as argued above, the operation of competitive forces.

In this Section, we therefore consider monopolistic competition environments in which symmetric firms produce a finite number of varieties $n$ which directly affects the equilibrium elasticity of demand.\textsuperscript{23} We first analyze the case of quasi-linear quadratic preferences, as in Melitz and Ottaviano (2008), then study the implications of strategic interaction à la Bertrand and Cournot, and finally discuss Lancaster (1979)'s ideal variety approach to monopolistic competition. Perhaps surprisingly, we find that none of the above setups seems to yield a robust pro-competitive mechanism in monopolistic competition.

4.1 Quasi-Linear Quadratic Preferences

Consider a D-S monopolistic competition setup with symmetric firms and a utility function $U = c_0 + u(c)$, where $c_0$ is consumption of an outside good, $c$ is the consumption vector

\textsuperscript{21}With non-additive preferences, the elasticity of substitution between any two varieties also depends on the consumption of other varieties: at a symmetric equilibrium, this translates into a direct impact of $n$ on the elasticity of substitution for a given level of consumption $c$ (see below).

\textsuperscript{22}See Pettengill (1979) on this point. He claims (p. 960) that "one’s normal idea of monopolistic competition is that as the number of products in the industry increases, they become closer and closer substitutes". He therefore argues that Lancaster’s (1975) ideal variety approach to monopolistic competition is more realistic in this respect.

\textsuperscript{23}Note that strategic interation cannot arise under the assumption, on which we relied in the previous Section, of a continuum of varieties.
of $n$ varieties of some product, and

$$u(c) = a \sum_{j=1}^{n} c_j - b \sum_{j=1}^{n} c_j^2 - \frac{\gamma}{2} \left( \sum_{j=1}^{n} c_j \right)^2,$$  \hspace{1cm} (26)

with $a, b, \gamma > 0$. The implied demand function for variety $i$ is

$$c_i = \frac{a}{b + n\gamma} - \frac{p_i}{b} + \frac{n\gamma}{b + n\gamma} \frac{1}{p},$$  \hspace{1cm} (27)

where $\bar{p} = \frac{1}{n} \sum_{j=1}^{n} p_j$ is the average price of a variety. Under the D-S assumption that firms take $n$ and $\bar{p}$ as given, firm $i$’s perceived demand is linear in $p_i$. Thus, at a symmetric equilibrium the perceived demand elasticity can be written as

$$\varepsilon = \frac{p}{bc} = \frac{1}{b} \left[ \frac{a}{c} - b - \gamma n \right].$$  \hspace{1cm} (28)

Note that, perhaps surprisingly, for given $c$ the number of firms $n$ has a negative direct impact on the elasticity of demand. This example shows that, even if non-additive preferences introduce a direct channel whereby $n$ affects $\varepsilon$, the resulting effect is not obviously positive.

Eq. (28) implies:

$$m(c) = 1 + \frac{bc}{\beta}, \quad p = m(c) \beta = bc + \beta.$$  \hspace{1cm} (29)

Using (29), the free-entry condition ($\pi_o = (m - 1) \beta L c = \alpha$) yields $m = 1 + \left( \frac{\sqrt{bc}}{L} \right) / \beta$, thereby implying that equilibrium markups are decreasing in market size (or, more precisely, in $L/\alpha$). However, given the negative direct impact of $n$ on $\varepsilon$ for given $c$, the pro-competitive effect of pure globalization is entirely driven by the fact that, just as in the case of DES preferences, the elasticity of substitution is decreasing in $c$.

Note also that $m$ is decreasing in $\beta$. In the Appendix we show that, as with DES preferences, this pass-through effect implies that, under costly trade, a reduction in the variable trade cost $\tau$ leads firms to charge lower markups in the domestic market and higher markups in the foreign market. \footnote{Similarly, when preferences are (homothetic) Translog, as in Feenstra (2003), it can be shown that a reduction in trade costs leads firms to charge higher markups in the foreign market. As shown by ACDR, this may lead to smaller welfare gains from trade opening than in the case of constant markups. However, Feenstra’s preferences imply that at a symmetric free trade equilibrium the elasticity of substitution is linearly increasing in $n$ and the markup is independent of the level of marginal cost $\beta$.} Moreover, a small increase in $\tau$ from the free trade leads to a reduction in average markups. We record these results in the following

\footnote{We assume that $c_0 > 0$, i.e., that an internal solution arises in equilibrium.}
Proposition 5  With quasi-linear quadratic preferences, as in the case of additive preferences, trade liberalization leads to opposite competitive effects in the domestic and foreign market, and average markups are non-monotonically related to trade costs.

4.2 Strategic Interaction

The Dixit-Stiglitz assumption that each firm treats the marginal utility of income, $\lambda$, as a constant removes a direct channel whereby an increase in the number of firms may raise the perceived demand elasticity $\varepsilon$. Instead, in a model with strategic interaction firms properly treat $\lambda$ as a function of prices, and the elasticity of demand no longer coincides with the elasticity of substitution.

Consider first a setting with additive preferences and Bertrand competition, in which firms set prices. In a symmetric Bertrand-Nash equilibrium, firms perceive the actual demand elasticity, which can be shown to equal

$$\varepsilon^b(c,n) = \sigma(c) - \frac{\sigma(c) - 1}{n}. \quad (30)$$

Note that $\partial \varepsilon^b / \partial n > 0$, which captures the direct pro-competitive channel induced by strategic interaction. Thus, the pricing rule is

$$p = m^b(c,n)\beta = \frac{(n - 1)\sigma(c) + 1}{(n - 1)\sigma(c)} m(c)\beta, \quad (31)$$

where $\partial m^b / \partial n < 0$, and $\partial m^b / \partial c \geq 0 \iff m' \geq 0 \iff \sigma' \leq 0$.

Next consider the case of Cournot competition, in which firms set production levels. As shown in the Appendix, in a symmetric Cournot-Nash equilibrium the pricing rule is

$$p = m^c(c,n)\beta = \frac{n}{n - 1} m(c)\beta, \quad (32)$$

where, once again, $\partial m^c / \partial n < 0$ and $\partial m^c / \partial c \geq 0 \iff m' \geq 0 \iff \sigma' \leq 0$. Accordingly, Cournotian firms perceive a demand elasticity equal to

$$\varepsilon^c(c,n) = \frac{\sigma(c)n}{\sigma(c) + n - 1}. \quad (33)$$

We can use the pricing rules, (31) or (32), the free-entry condition ($p = \beta + \alpha/cL$) and the budget constraint ($npc = 1$) to characterize the symmetric Bertrand and Cournot

---

26Note that $\varepsilon^c(c,n) < \varepsilon^b(c,n) < \varepsilon(c)$, i.e., with additive preferences Cournotian firms perceive a lower demand elasticity relative to Bertrand firms, which in turn perceive a lower elasticity than under the Dixit-Stiglitz assumption.
equilibria, summarized by the two-equation system

\[ m^s(c^s, n^s)c^s \beta = \frac{1}{n^s} = \frac{\alpha}{L} + c^s\beta, \]

where \( s = b, c \). As shown in the Appendix, differentiating (34) and applying Cramer’s rule yields:

\[ \frac{\partial c^s}{\partial (\alpha/L)} \geq 0, \quad \frac{\partial c^s}{\partial \beta} < 0, \quad \frac{\partial n^s}{\partial (\alpha/L)} < 0 \text{ and } \text{sign} \left\{ \frac{\partial n^s}{\partial \beta} \right\} = \text{sign} \left\{ \sigma' \right\}, \]

where \( \frac{\partial c^s}{\partial (\alpha/L)} = 0 \) only for \( s = c \) and \( n = 2 \). This allows us to study the competitive effects of pure globalization (a reduction in \( \alpha/L \)) and of a reduction in the marginal cost \( \beta \). Note first that

\[ \frac{\partial m^s}{\partial (\alpha/L)} = \frac{\partial m^s}{\partial c} \frac{\partial c^s}{\partial (\alpha/L)} + \frac{\partial m^s}{\partial n} \frac{\partial n^s}{\partial (\alpha/L)}, \]

where \( \frac{\partial m^s}{\partial n} < 0 \) and (35) imply that the second term on the RHS is always positive. It follows that \( \frac{\partial m^s}{\partial (\alpha/L)} > 0 \) for \( \sigma' \leq 0 \), as in this case \( \frac{\partial m^s}{\partial c} \geq 0 \). Consequently, an increase in market size is pro-competitive when preferences are DES or CES. Instead, as shown in the Appendix, \( \frac{\partial m^s}{\partial (\alpha/L)} < 0 \) for \( \sigma' > 0 \), unless \( n \) is small. Thus, pure globalization is generally anti-competitive with IES preferences.

Finally, note that

\[ \frac{\partial m^s}{\partial \beta} = \frac{\partial m^s}{\partial c} \frac{\partial c^s}{\partial \beta} + \frac{\partial m^s}{\partial n} \frac{\partial n^s}{\partial \beta}, \]

implying that the change in individual consumption and in the number of firms induced by a change in the marginal cost \( \beta \) affect markups in opposite directions. In the Appendix we show, however, that in all cases \( \text{sign} \left\{ \frac{\partial m^s}{\partial \beta} \right\} = \text{sign} \left\{ \sigma' \right\} \), and hence that a reduction in the marginal cost is anti-competitive with DES preferences, and pro-competitive with IES preferences. We summarize our main results in the following

**Proposition 6** When firms interact strategically, pure globalization is pro-competitive with DES or CES preferences; with IES preferences, instead, a pro-competitive effect may arise only when the number of firms is small. Moreover, a reduction in the marginal cost is anti-competitive with DES preferences, neutral with CES preferences, and pro-competitive with IES preferences.

4.2.1 Discussion

With or without strategic interaction, additive non-CES preferences seem to yield the same competitive effects. In particular, pure globalization is generally anti-competitive

\[ \text{\footnotesize \cite{footnote}} \]

\[ \text{\footnotesize \cite{footnote2}} \]
when preferences are IES. Moreover, the competitive effects of a reduction in the marginal cost are generally the opposite than those of an increase in market size. This implies, e.g., that with DES preferences markups increase when marginal costs fall, and thus the pro-competitive effect of a reduction in trade costs would still be contaminated by the increase in markups on foreign sales due to an incomplete pass-through.

Finally note, from (30) and (33), that the competitive effects arising from strategic interaction can be relevant only when the number of firms is small, or else the impact on markups of an increase in $n$ becomes negligible. However, as forcefully argued by Dixit and Stiglitz (1993), assuming that $n$ is small is problematic in a monopolistic competition setting. In particular, if the number of firms is small enough to induce them to interact strategically, it is unclear why they do not also engage in collusion and entry deterrence, thereby preventing the free entry of firms. By the same token, it is actually unclear why we should expect significant competitive effects in a market situation in which the number of firms is large enough to make sense of the free-entry condition postulated in monopolistic competition models.

4.3 Ideal Variety Approach to Monopolistic Competition

Finally, one may argue that a pro-competitive effect may naturally arise in a framework in which an increase in the number of available varieties reduces their distance in a fixed space of characteristics, thereby increasing their substitutability. In closing this Section we show that, surprisingly, this need not be the case.

To make the point, we reconsider Lancaster’s (1975, 1979) ideal variety approach to monopolistic competition. In this setting, each consumer has a most preferred ("ideal") variety, and the market demand for each variety arises from the diversity of tastes. Formally, each variety is represented by a point $\omega$ on the unit length circumference $\Omega$ of a circle, and preferences for the ideal product within a mass $L$ of consumers are uniformly distributed over $\Omega$. The utility function of a consumer with ideal variety $\tilde{\omega}$ is assumed to be:

$$ U = \sum_{\omega \in \Omega} \frac{c(\omega)}{h(\delta(\omega, \tilde{\omega}))}, \quad (38) $$

where $\delta(\omega, \tilde{\omega})$ is the shortest arc distance between $\omega$ and $\tilde{\omega}$, and $h(\delta)$ is the so-called Lancaster’s compensation function, assumed to be positive, non-decreasing and generally

---

28As described in Helpman and Krugman (1985, pp. 120-21), ideal variety means that "when the individual is offered a well-defined quantity of the good but is free to choose any potentially possible variety, he will choose the ideal variety independently of the quantity offered and independently of the consumption level of other goods. Moreover, when comparing a given quantity of two different varieties, the individual prefers the variety that is closest to his ideal product."
normalized so that \( h(0) = 1 \) (see Lancaster, 1975). Moreover, it is generally assumed (see Helpman and Krugman, 1985) that \( h(\delta) \) is strictly increasing and convex, and that \( h'(0) = 0 \).

The above assumptions imply that the marginal rate of substitution between \( \tilde{\omega} \) and \( \omega \) is given by \( MRS(\omega, \tilde{\omega}) = h(\delta(\omega, \tilde{\omega})) \), and is thus an increasing convex function of \( \delta \). Following Helpman (1981) and Helpman and Krugman (1985), it is possible to show that, at a symmetric equilibrium, the price elasticity of market demand equals

\[
\varepsilon = 1 + \frac{1}{2e_h(1/2n)},
\]

where \( e_h = \frac{h'(1/2n)}{2nh(1/2n)} \) is the elasticity of the compensation function. Accordingly, an increase in market size \( \mathcal{L} \), by increasing the number of firms \( n \), also increases \( \varepsilon \) if \( e'_h > 0 \), thus yielding a pro-competitive effect.\(^{29}\) Instead, an increase in market size reduces \( \varepsilon \) and is therefore anti-competitive if \( e'_h < 0 \). Finally, if \( e'_h = 0 \) (i.e., if \( h(\cdot) \) is isoelastic), \( \varepsilon \) is independent of \( n \), just as in the D-S "love for variety" approach when preferences are CES. Thus, a pro-competitive effect is unwarranted even in a framework in which an increase in the number of firms crowds the variety space. There are two main reasons for this result. First, given (38), varieties are of the "perfect substitute" type, their elasticity of substitution is constant, and a crowding of the variety space cannot affect it. In this respect, the model fails to capture a potentially genuine pro-competitive effect of the crowding of the variety space.

Second, the ideal variety approach does not impose sufficient restrictions on \( h(\cdot) \) to pin down the properties of \( e_h \), and therefore the relationship between \( \varepsilon \) and \( n \).\(^{30}\) Specifically, is the assumption \( e'_h > 0 \) plausible? Note that \( MRS(\tilde{\omega}, \omega) = h(\delta(\omega, \tilde{\omega})) \) implies that, in order for Lancaster’s model to deliver a pro-competitive effect, consumer preferences must feature an ever increasing distance elasticity of the marginal rate of substitution between \( \tilde{\omega} \) and \( \omega \). It is hard to provide a rationale for this assumption, which seems no more plausible than the opposite assumption of a decreasing distance elasticity, which would however deliver an anti-competitive effect. We summarize in the following

**Proposition 7** When preferences are of the ideal variety type, as in Lancaster (1979), pure globalization does not affect markups if the compensation function \( h(\cdot) \) is isoelastic;

\(^{29}\)For instance, in a generalized model of ideal variety with income effects, Hummels and Lugovskyy (2012) assume, without discussion, a functional form for the compensation function that implies \( e'_h > 0 \) and thus delivers pro-competitive effects.

\(^{30}\)Only in the limit case in which \( n \) goes to infinity, and due to the (rather ad hoc) assumptions \( h(0) = 1 \) and \( h'(0) = 0 \), the elasticity of market demand is increasing in \( n \) (and goes to infinite). This requires a situation, unfeasible under a positive fixed cost, in which the circumference of the circle is full and the demand for each firm is infinitesimal.
it is instead pro-competitive when $h(\cdot)$ features an increasing distance elasticity, and anti-competitive when the distance elasticity is decreasing.

5 Conclusion

We have studied the competitive and reallocation effects of trade opening in monopolistic competition. We have shown that, in a Dixit-Stiglitz setup with additively separable preferences, heterogeneous firms and fixed and variable costs of exporting, the extensive margin (i.e., Melitz-type selection) effects of trade opening are robust to relaxing the CES assumption, whereas the intensive margin effects (i.e., market share reallocations across inframarginal firms) are fragile. Moreover, measured productivity gains from trade opening are not ensured in this setup. We have also argued that these results arise from the fragility of competitive (i.e., markup) effects, and that allowing for non-additive preferences, for strategic interaction and for the crowding of the variety space does not obviously lead to stronger and more robust competitive effects in monopolistic competition.

Our results suggest that the effects of trade opening delivered by non-CES preferences in monopolistic competition are more complex and articulated than generally believed. Consistently, one should carefully look for empirical evidence in support, e.g., of the implied anti-competitive effects. Alternatively, one can invoke a sort of second-best argument in support of CES preferences, according to which relaxing one out of a number of restrictive assumptions does not necessarily lead to a better model. We think that the latter option is not equivalent to assuming that competitive effects are little relevant, but rather that they should be studied in different trade models. In this respect, oligopoly settings seem a prominent alternative, as suggested by Neary (2003, 2009b, 2010), who also warns us about the difficulties of dealing with them in general equilibrium. We conclude that more creative theoretical work is still needed to better our understanding of the competitive and reallocation effects of globalization.

6 Appendix

6.1 Proof of Results in Section 2

Lemma 1. We begin by proving Lemma 1. Note first that, differentiating (9) with respect to $L$ and using (8), yields:

$$1 + \frac{\partial \ln \pi_v(\beta^*, c^*, L)}{\partial \ln c} \frac{\partial \ln c^*}{\partial \ln L} = 1 + \frac{\sigma(c^*)}{\eta(c^*)} \frac{\partial \ln c^*}{\partial \ln L} = 0.$$
Thus,
\[
\frac{\partial \ln \alpha}{\partial \ln L} = \frac{\eta(c^*)}{\sigma(c^*)} = \frac{\partial \ln \alpha}{\partial \ln \beta^*} = -\frac{\partial \ln \beta^*}{\partial \ln \alpha} < 0. \tag{39}
\]
Next, differentiating (10) with respect to \(\beta^*\) gives:
\[
r''(c) \frac{\partial \beta}{\partial \beta^*} = \frac{\beta}{\beta^2} \left[ r''(c^*) \frac{\partial c^*}{\partial \beta^*} - r'(c^*) \right].
\]
Rearranging terms and using (39), we obtain:
\[
\frac{\partial \ln c^*}{\partial \ln \beta^*} = \frac{\eta(c)}{m(c^*)} > 0.
\]
Similarly, differentiating (10) with respect to \(L\) and using (39) yields:
\[
\frac{\partial \ln c}{\partial \ln L} = -\frac{\eta(c)}{\sigma(c^*)} = -\frac{\partial \ln c}{\partial \ln \alpha} < 0,
\]
which completes our proof of Lemma 1.

Firm size, profits and markups. We now prove the results in equations (13)-(15). Differentiating (4) gives:
\[
r''(c(\beta)) \frac{dc(\beta)}{d\beta} = \lambda = \frac{r'(c)}{\beta}.
\]
Rearranging terms yields:
\[
\frac{d \ln c(\beta)}{d \ln \beta} = \frac{r'(c(\beta))}{r''(c(\beta))c(\beta)} = -\frac{\eta(c(\beta))}{\eta(c(\beta))} < 0. \tag{40}
\]
As for the elasticity of markups with respect to \(\beta\), using (6) and (40), we obtain:
\[
\frac{d \ln m(\beta)}{d \ln \beta} = \frac{d \ln m}{d \ln c} \frac{d \ln c(\beta)}{d \ln \beta} = -\left( \frac{1}{\eta(c(\beta))} - \frac{1}{\sigma(c(\beta))} \right) \eta(c(\beta)) = \frac{\eta(c(\beta))}{\sigma(c(\beta))} - 1.
\]
Next, recalling that \(p(\beta)c(\beta) = m(c(\beta))\beta c(\beta)\), and using the above results, we obtain:
\[
\frac{d \ln \{p(\beta)c(\beta)\}}{d \ln \beta} = \frac{d \ln m(\beta)}{d \ln \beta} + 1 + \frac{d \ln c(\beta)}{d \ln \beta} = \frac{\eta(c(\beta))}{\sigma(c(\beta))} - \eta(c(\beta))
\]
\[
= 1 - \left[ 1 + \eta(c(\beta)) \left( 1 - \frac{1}{\sigma(c(\beta))} \right) \right] = 1 - \chi(c(\beta)) < 0.
\]
Note also that \(\chi \geq \sigma\) is equivalent to \(\sigma - \eta + \sigma\eta - \sigma^2 \geq 0\). Recalling that \(\sigma'(c) \geq 0 \iff \eta(c) \geq \sigma(c)\), it follows that, \(\chi(c) \geq \sigma \iff \sigma'(c) \geq 0\).
Finally, using (8) and (40), the elasticity of variable profits can be written as:

\[
\frac{d \ln \pi_v(\beta)}{d \ln \beta} = 1 + \frac{\partial \ln \pi_v}{\partial \ln c} \frac{d \ln c}{d \ln \beta} = 1 - \sigma(c(\beta)) < 0.
\]

This completes our proof of the results in (13)-(15).

6.2 Proof of Results in Section 3

6.2.1 Pure Globalization

**Extensive margin effects.** The impact of an increase in market size \(L\) on the marginal cost cutoff \(\beta^*\) can be computed by applying the implicit function theorem to (11):

\[
\frac{d\beta^*}{dL} = -\frac{\partial \pi^E}{\partial L} \frac{\partial \pi^E}{\partial \beta^*}.
\]

Note that \(\frac{\partial \pi^E}{\partial \beta^*} > 0\) by (12). As for the term on the numerator, differentiating the free-entry condition (11) with respect to \(L\), using (8) and Lemma 1, yields:

\[
\frac{\partial \ln \pi^E}{\partial \ln L} = \int_{\beta}^{\beta^*} \left( 1 + \frac{\partial \ln \pi_v}{\partial \ln c} \frac{\partial \ln c}{\partial \ln L} \right) dG(\beta) = \int_{\beta}^{\beta^*} \left( 1 - \frac{\sigma(c(\beta))}{\sigma(c^*)} \right) dG(\beta).
\]

Note that, for \(c^* < c\), \(\sigma(c^*) > \sigma(c)\) with DES preferences, and \(\sigma(c^*) < \sigma(c)\) with IES preferences, thereby implying that \(\frac{\partial \ln \pi^E}{\partial \ln L} \leq 0 \Leftrightarrow \sigma' \geq 0\). Consequently, \(\frac{\partial \beta^*}{\partial L} \geq 0 \Leftrightarrow \sigma' \geq 0\). This proves that the selection/anti-selection effects of pure globalization are determined by the sign of \(\sigma'\).

**Effects on measured productivity.** Consider the "average marginal cost" \(\tilde{\beta}\) implicitly defined by the following property of total cost invariance:

\[
n \left[ \int_{\beta}^{\beta^*} \tilde{\beta} c(\beta) L \frac{dG(\beta)}{G(\beta^*)} + \alpha \right] = n \left[ \int_{\beta}^{\beta^*} \beta c(\beta) L \frac{dG(\beta)}{G(\beta^*)} + \alpha \right].
\]

It is easily seen that

\[
\tilde{\beta} = \int_{\beta}^{\beta^*} \beta d\Gamma(\beta),
\]

where

\[
\Gamma(\beta) = \int_{\beta}^{\beta^*} \frac{c(z) dG(z)}{\int_{\beta}^{\beta^*} c(s) dG(s)}
\]

23
is the distribution of $\beta$ weighted by the corresponding production levels. A measure of aggregate productivity is therefore provided by $1/\beta$. Notice that $\beta$ is determined by $\Gamma(\beta)$, which has support $[\underline{\beta}, \overline{\beta}]$ and depends on $G(\beta)$ and $c(\beta)$.

A simple "stochastic dominance" argument implies that, when $\eta'$ and $\sigma'$ agree in sign, we can predict the impact of market size on aggregate productivity. Let $\gamma = \Gamma'$ be the density function associated with $\Gamma$; denote by $h = d\log \Gamma/d\beta = \gamma/\Gamma$ the corresponding so-called reverse hazard rate, and recall that, if two distributions have the same reverse hazard rate, they are identical. We obtain:

$$h(\beta) = \frac{c(\beta)g(\beta)}{\int_{\beta} c(z)dG(z)} = \frac{g(\beta)}{\int_{\beta} \frac{c(z)}{c(\beta)}dG(z)},$$

which implies that the reverse hazard rate corresponding to a $\beta$-firm depends on its output relative to that of all other firms with a lower marginal cost. Thus, by governing the relative output of inframarginal firms, $\eta$ affects how $L$ impacts on $\beta$. However, pure globalization affects measured productivity also through the extensive margin effects.

In particular, consider an increase in market size from $L$ to $\hat{L}$, and suppose that $\eta', \sigma' < 0$. Denote by $\Gamma_L$ and $\hat{\Gamma}_L$ the corresponding distributions, with supports $[\overline{\beta}, \overline{\beta}_L]$ and $[\overline{\beta}, \overline{\beta}_L]$ respectively, where $\overline{\beta}_L < \overline{\beta}$ due to the selection effect implied by DES preferences. For all $\beta \in [\overline{\beta}, \overline{\beta}_L]$ (i.e., for all inframarginal firms), $\eta' < 0$ implies

$$\int_{\overline{\beta}}^{\beta} \frac{c_L(z)}{c_L(\beta)}dG(z) \leq \int_{\overline{\beta}}^{\beta} \frac{c_{\hat{L}}(z)}{c_L(\beta)}dG(z),$$

and then $h_{\hat{L}}(\beta) \leq h_L(\beta)$ (an intensive margin effect). Moreover, since $h_{\hat{L}}(\beta) = 0 \leq h_L(\beta)$ for all $\beta \in (\overline{\beta}_L, \overline{\beta}_L)$ (due to the extensive margin effect), it follows that $h_{\hat{L}}(\beta) \leq h_L(\beta)$ for all $\beta \in [\overline{\beta}, \overline{\beta}_L]$. But then $\Gamma_L$ is "reverse-hazard rate dominated" by $\Gamma_L$ (see, e.g., Shaked and Shanthikumar, 1994), and thus $\overline{\beta}_L < \overline{\beta}_L$, implying an increase in measured productivity. By a similar reasoning, if $\eta', \sigma' > 0$, a rise in market size leads to an unambiguous reduction in measured productivity.

On the contrary, if $\eta'$ and $\sigma'$ do not agree in sign, the selection effect (governed by $\sigma$) and the intensive margin effect in terms of firm output (governed by $\eta$) point in opposite directions. In this case, the impact of pure globalization on measured productivity becomes ambiguous, as it will in general depend on the properties of the distribution $G$. 

24
6.2.2 Costly Trade

**Extensive margin effects.** Consider first a reduction in the variable trade cost $\tau$. Applying the implicit function theorem to (20) yields:

$$\frac{\partial \ln \beta^*}{\partial \ln \tau} = -\frac{\partial \pi^E / \partial \tau}{\partial \pi^E / \partial \beta^*}.$$

(41)

Noting that $d\pi(\beta)/d\beta = -c(\beta)L$ by the Envelope Theorem, and recalling that $\pi_x(\beta^*_x) = 0$, we obtain:

$$\frac{\partial \pi^E}{\partial \tau} = \int^{\beta^*_x}_\beta \frac{\partial \pi_v(\tau \beta)}{\partial \tau} dG(\beta) = -L \int^{\beta^*_x}_\beta c(\tau \beta) \beta dG(\beta) < 0.$$

Similarly, noting that $\partial \pi_v / \partial \beta = \beta \mu(c)/c$ by (7)and (8), using Lemma 1, and also recalling that $\pi(\beta^*) = 0$, gives:

$$\frac{\partial \pi^E}{\partial \beta^*} = \int^{\beta^*_x}_\beta \frac{\partial \pi_v}{\partial \beta} dG(\beta) = \int^{\beta^*_x}_\beta \frac{\partial \pi_v}{\partial \beta} \frac{\partial c}{\partial \beta^*} dG(\beta)$$

$$= \frac{L}{m(c^*)} \left[ \int^{\beta^*_x}_\beta p(\beta)c(\beta)dG(\beta) + \int^{\beta^*_x}_\beta p(\tau \beta)c(\tau \beta)dG(\beta) \right] > 0.$$

Using the above results in (41) gives (21), thereby proving that $\beta^*$ is increasing in $\tau$.

Consider now a reduction in the fixed cost of exporting $\alpha_x$. Applying the implicit function theorem to (20) yields:

$$\frac{\partial \beta^*}{\partial \alpha_x} = \frac{\partial \pi^E / \partial \alpha_x}{\partial \pi^E / \partial \beta^*},$$

where $\partial \pi^E / \partial \beta^* > 0$ and

$$\frac{\partial \pi^E}{\partial \alpha_x} = \int^{\beta^*_x}_\beta \frac{\partial \pi_x(\beta)}{\partial \alpha_x} dG(\beta) = -G(\beta^*_x) < 0,$$

implying that $\partial \beta^* / \partial \alpha_x > 0$. This proves that $\beta^*$ is increasing in $\alpha_x$.

6.3 Proof of Results in Section 4

6.3.1 Quasi-Linear Quadratic Preferences

We now prove the results in Proposition 5. To this purpose, consider two identical countries separated by a symmetric iceberg trade cost $\tau \geq 1$. Indexing variables related to the
foreign market by an $x$ and using (29) gives:

$$m_x(c_x) = 1 + \frac{b c_x}{\beta \tau}, \quad p_x = m_x(c_x) \tau \beta = bc_x + \tau \beta. \quad (42)$$

To compute the free-entry condition ($\pi_v + \pi_v^x = \alpha$), note first that, using (29) and (42), variable profits in the domestic and foreign market can be written as:

$$\pi_v(c) = [m(c) - 1] \beta L c = b L c^2, \quad (43)$$
$$\pi_v^x(c_x) = [m_x(c_x) - 1] \tau \beta L c_x = b L c_x^2 = \pi_v(c_x).$$

Next, note that (27) and the expressions for $p$ and $p_x$ in (29) and (42) imply that

$$c - c_x = \frac{p_x - p}{b} = \frac{\beta (\tau - 1)}{2b}, \quad (44)$$

which defines $c_x$ as a function of $c$ and $\tau$, i.e., $c_x(c, \tau) = c - \beta (\tau - 1)/2b$. Using (43) and (44) in the free-entry condition, which implicitly defines $c$, yields:

$$\pi_v(c) + \pi_v(c_x(c, \tau)) = b L c^2 + b L \left( c - \frac{(\tau - 1) \beta}{2b} \right)^2 = \alpha. \quad (45)$$

Differentiating (44) and (45) yields:

$$\frac{\partial c}{\partial \tau} = \frac{\beta}{2b} \frac{c_x}{c + c_x} > 0, \quad \frac{dc_x}{d\tau} = \frac{\beta}{2b} \left( \frac{c_x}{c + c_x} - 1 \right) < 0.$$

Thus, using (29) and (42):

$$\frac{dm}{d\tau} = \frac{b}{\beta} \left( \frac{dc}{d\tau} \right), \quad \frac{dm_x}{d\tau} = \frac{b}{\beta^2} \left( \frac{dc_x}{d\tau} \tau - c_x \right) = \frac{1}{2\tau} \left( \frac{c_x}{c + c_x} - 1 \right) - \frac{bc_x}{\beta \tau^2} < 0.$$

Consequently, after a reduction in trade costs, firms charge lower markups in the domestic market and higher markups in the foreign market.

Finally, computing the above derivatives in $\tau = 1$ yields:

$$\frac{\partial c}{\partial \tau} \bigg|_{\tau=1} = \frac{\beta}{4b} = -\frac{dc_x}{d\tau} \bigg|_{\tau=1},$$
$$\frac{dm}{d\tau} \bigg|_{\tau=1} = \frac{1}{4} + \frac{bc}{\beta} > \frac{1}{4} = \frac{dm_x}{d\tau} \bigg|_{\tau=1}.$$

Thus, a small increase in trade costs around the free trade causes markups on foreign sales
to fall by more (in absolute value) than the increase in markups on domestic sales, thus leading to a reduction in average markups.

6.3.2 Comparative Statics of the Bertrand and Cournot Equilibria.

We now prove the results in Proposition 6. Note first that, differentiating (34), yields:

$$
\begin{bmatrix}
\frac{\partial m^s}{\partial c} c^s + m^s \\
\beta
\end{bmatrix}
\begin{bmatrix}
\frac{\partial m^s}{\partial n^s} \\
1/(n^s)^2
\end{bmatrix}
\begin{bmatrix}
\frac{\partial e^s}{\partial (\alpha/L)} \\
\frac{\partial m^s}{\partial (\alpha/L)}
\end{bmatrix}
\begin{bmatrix}
0 \\
-1
\end{bmatrix}
\begin{bmatrix}
-m^se^s \\
-c^s
\end{bmatrix}.
$$

(46)

Thus, applying Cramer’s rule,

$$
\begin{bmatrix}
\frac{\partial e^s}{\partial (\alpha/L)} \\
\frac{\partial m^s}{\partial (\alpha/L)}
\end{bmatrix}
= \frac{1}{D^s}
\begin{bmatrix}
\frac{\partial m^s}{\partial n^s} c^s + m^s + \frac{1}{(n^s)^2} \\
-\left(\frac{\partial m^s}{\partial c} c^s + m^s\right) \beta
\end{bmatrix}
\begin{bmatrix}
\frac{\partial m^s}{\partial (\alpha/L)} c^s + \frac{1}{(n^s)^2} \\
\frac{\partial m^s}{\partial (\alpha/L)}
\end{bmatrix}
\begin{bmatrix}
0 \\
-1 \\
-c^s
\end{bmatrix},
$$

(47)

where $D^s$ is the determinant of the first matrix on the LHS of (46).

**Bertrand competition.** Differentiating (31) yields:

$$
\begin{align*}
\frac{\partial m^b}{\partial c} &= \frac{n}{n-1} m', \\
\frac{\partial m^b}{\partial n} &= -\frac{1}{(n-1)^2 (\sigma - 1)} = -\frac{m}{(n-1)^2 \sigma} = -\frac{m^b}{(n-1)(n-1)\sigma + 1}.
\end{align*}
$$

(48)

Next, using (48) in (47) yields:

$$
D^b
\begin{bmatrix}
\frac{\partial e^b}{\partial (\alpha/L)} \\
\frac{\partial m^b}{\partial (\alpha/L)}
\end{bmatrix}
= \begin{cases}
1 \left(1 - \frac{m}{(n^b)^2 (n^b - 1)^2 m^b}\right) & \text{if } c > 0 \\
\frac{c}{(n^b)^2} \left(1 - \frac{m^b}{(n^b - 1)((n^b - 1)\sigma + 1)}\right) & \text{if } c < 0
\end{cases}
\begin{bmatrix}
\frac{\partial e^b}{\partial (\alpha/L)} \\
\frac{\partial m^b}{\partial (\alpha/L)}
\end{bmatrix}
\begin{cases}
> 0 \\
< 0
\end{cases}
\begin{bmatrix}
\frac{\beta}{n^b - 1} \left(n^b (m' c^b + m) - 1\right) \\
-\frac{\beta}{n^b - 1} m'
\end{bmatrix},
$$

(49)

where

$$
D^b = \frac{\beta}{n^b (n^b - 1)} \left[m' c^b + m - 1 + \frac{m}{\sigma (n^b - 1) m^b}\right] > 0,
$$

and the inequalities follow from $\pi'_v(c) > 0$ (which implies $m' c + m - 1 > 0$) and the definition of $m^b$. This proves that the inequalities in (35) hold for $s = b$. 

27
Next, using (49) in (36) yields:

\[
\frac{\partial m^b}{\partial (\alpha/L)} = \frac{n^b m' \partial c^b}{n^b - 1} - \frac{m^b}{(n^b - 1) [(n^b - 1) \sigma + 1]} \frac{\partial n^c}{\partial L}
\]

\[
= \frac{1}{D^b} \left\{ \frac{m'}{n^b - 1} \left[ 1 - \frac{m^b}{(n^b - 1) \sigma (n^b - 1)^2 m^b} + \frac{m^b \beta [(n^b - 1)^2 (n^b - 1) \sigma + 1] - m(n^b - 1) \sigma (n^b - 1)^2 m^b]}{(n^b - 1)^2 (n^b - 1) \sigma (n^b - 1) + 1] D^b \right) \right\}
\]

where \( g(n) = [(n - 1)/n] [(n - 1) \sigma + 1] > 0 \) is a monotonically increasing function, with \( g(1 + 1/\sqrt{\sigma}) = 1 \). Note that \( \partial m^b/\partial (\alpha/L) \) > 0 if and only if \( m + m' c^b g(n^b) > 1/n^b \), a condition that is always satisfied for \( m' \geq 0 \) (i.e., with DES or CES preferences). Instead, for \( m' < 0 \) (IES preferences), \( \partial m^b/\partial (\alpha/L) < 0 \) unless \( n^c \) is small, i.e., sufficiently close to \( 1 + 1/\sqrt{\sigma} \).

Finally, using (49) in (37) yields:

\[
\frac{\partial m^b}{\partial \beta} = \frac{m'}{n^b - 1} \frac{n^b \partial c^b}{\partial \beta} - \frac{m^b}{(n^b - 1) [(n^b - 1) \sigma + 1]} \frac{\partial n^b}{\partial \beta}
\]

\[
= \frac{1}{(n^b - 1) D^b} \left\{ m' \frac{c^b}{n^b} \left( 1 - m^b - \frac{n^b}{(n^b - 1) \sigma (n^b - 1) + 1}] \right) + \frac{m^b m' \beta (c^b)^2}{(n^b - 1)^2 (n^b - 1) \sigma (n^b - 1) + 1] D^b \right) \right\}
\]

which implies that \( \text{sign} \left\{ \frac{\partial n^c}{\partial \beta} \right\} = -\text{sign} \left\{ m' \right\} = \text{sign} \left\{ \alpha' \right\} \).

Cournot competition. Using \( \lambda = \sum_j u'(c_j) \) in the first-order conditions for utility maximization \( u'(c_i) = \lambda p_i \) yields the following inverse demand system:

\[
p_i(c) = \frac{u'(c_i)}{\sum_j u'(c_j)c_j}, \quad i = 1, \ldots, n.
\]

(50)

Firm \( i \)'s revenue is \( p_i(c) \) \( c_i \); marginal revenue is therefore given by

\[
\frac{\partial (p_i(c) \) \) / \partial c_i = \frac{r''(c_i) \left( \sum_{j \neq i} u'(c_j) \right) \left( \sum_{j \neq i} u'(c_j)c_j \right)^2}{\left( \sum_{j \neq i} u'(c_j)c_j \right)^2},
\]

and is decreasing in \( c_i \) under our assumptions that \( r'' < 0 \) and \( r' > 0 \). In a Cournot-Nash equilibrium, each firm chooses its quantity to satisfy the first-order condition \( \partial (p_i(c) \) / \partial c_i = \beta \) under a correct conjecture about the quantities produced by its competitors. Then (51)
implies that, in any symmetric Cournot-Nash equilibrium,

\[ c = \frac{(n - 1)r'(c)}{n^2 u'(c) \beta} = \frac{n - 1}{n^2 m(c) \beta}. \tag{52} \]

Using (50) and (52) yields equation (32) in the main text.

Differentiating (32) yields:

\[ \frac{\partial m^c}{\partial c} = \frac{n}{(n - 1)^2} m', \quad \frac{\partial m^c}{\partial n} = -\frac{m}{n(n - 1)} = -\frac{m^c}{n(n - 1)}. \tag{53} \]

Next, using (53) in (47) yields:

\[ D^c \begin{bmatrix} \frac{\partial c^e}{\partial (\alpha/L)} & \frac{\partial c^e}{\partial \sigma} \\ \frac{\partial c^e}{\partial (\alpha/L)} & \frac{\partial c^e}{\partial \beta} \end{bmatrix} = \begin{cases} \frac{n^c - 2}{(n^c - 1)^2} \quad & \frac{c^e}{n^c (n^c - 1)} \left(1 - m - \frac{2}{n^c}\right) \\ \frac{n^c \beta}{n^c - 1} (m'c^e + m) & -\frac{n^c \beta (c^e)^2}{n^c - 1} m' \end{cases}, \tag{54} \]

where

\[ D^c = \frac{\beta}{(n^c)^2} \left[ \frac{n^c (m'c^e + m)}{n^c - 1} - 1 + \frac{1}{n^c - 1} \right] > 0. \tag{55} \]

Note that \( \partial c^e/\partial (\alpha/L) = 0 \) only for \( n^c = 2 \), and that the inequalities in (54) and (55) are straightforward once recalling that \( m'c + m - 1 > 0 \). Thus, the inequalities in (35) hold also for \( s = c \).

Next, using (54) in (36) yields:

\[ \frac{\partial m^c}{\partial (\alpha/L)} = \frac{n^c m'}{n^c - 1} \frac{\partial c^e}{\partial (\alpha/L)} - \frac{m}{(n^c - 1)^2} \frac{\partial n^c}{\partial (\alpha/L)} = \frac{m + m'c^e(n^c - 1)}{n^c (n^c - 1)^2} D^c c^e. \]

Note that \( \partial m^c/\partial (\alpha/L) > 0 \) if and only if \( m + m'c^e(n^c - 1) > 0 \), a condition that is always satisfied for \( m' \geq 0 \) (i.e., with DES or CES preferences). Instead, for \( m' < 0 \) (IES preferences), \( \partial m^c/\partial (\alpha/L) < 0 \) unless \( n^c \) is small, i.e., sufficiently close to 2.

Finally, using (54) in (37) yields:

\[ \frac{\partial m^c}{\partial \beta} = \frac{n^c m'}{n^c - 1} \frac{\partial c^e}{\partial \beta} - \frac{m c^e}{n^c (n^c - 1)} \frac{\partial n^c}{\partial \beta} = \frac{c^e m' (1 - m - \frac{1}{n^c})}{(n^c - 1)^2} D^c c^e, \]

implying that \( \text{sign} \left\{ \frac{\partial m^c}{\partial \beta} \right\} = -\text{sign} \left\{ m' \right\} = \text{sign} \left\{ \sigma' \right\} \). This completes our proof of Proposition 6.
References


